Turbulent Transfer Processes in the Atmospheric Surface Layer and their Parameterization for Strongly Stable Stratification

Gerhard Kramm[1] & Nicole Mölders[1,2]

University of Alaska Fairbanks, [1]Geophysical Institute and [2]College of Natural Science and Mathematics

Energy flux balance at the earth's surface

$$R_B - L_v(T_G)\,Q - H + G = 0$$

R_B = radiation flux balance
Q = water vapor flux
H = sensible heat flux
G = ground heat flux

$$Q = -\bar{\rho}\, u_* \, q_*$$

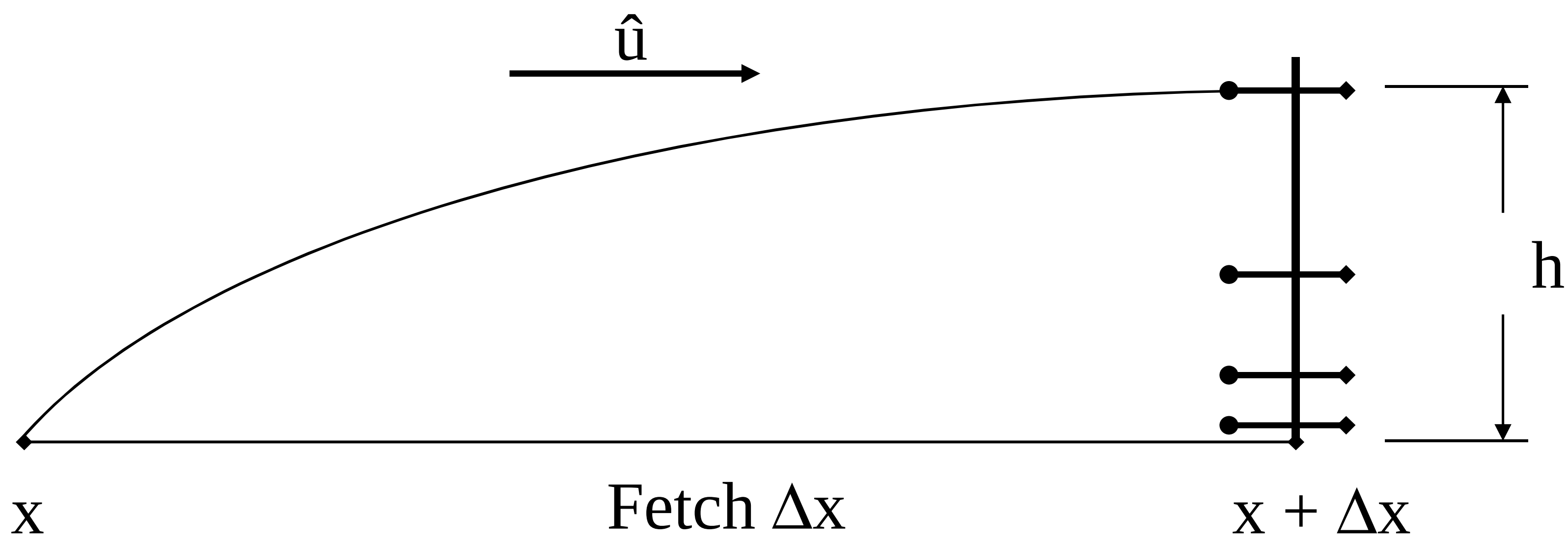
$$H = -c_{p,0}\, \bar{\rho}\, u_* \, \Theta_*$$

$$|\tau| = \bar{\rho}\, u_*^{\,2}$$

τ = friction stress vector

Assuming:

- Steady-state conditions,
- Horizontally homogeneous conditions,
- No phase transition processes



$$\frac{h}{\Delta x} \approx 0.010 \cdots 0.025$$

Hesselberg vs. Reynolds

Hesselberg

$$\frac{\partial \bar{\rho}}{\partial t} + \nabla \cdot \left(\bar{\rho}\, \hat{\mathbf{v}} \right) = 0$$

$$\bar{p} \cong \bar{\rho}\, R_0\, \hat{T}_v$$

$$\frac{1}{2} \overline{\rho\, \mathbf{v}^2} = \frac{1}{2} \bar{\rho}\, \hat{\mathbf{v}}^2 + \frac{1}{2} \overline{\rho\, \mathbf{v}''^2}$$

$$\frac{d\hat{\varphi}}{dt} = \frac{\partial \hat{\varphi}}{\partial t} + \hat{\mathbf{v}} \cdot \nabla \hat{\varphi}$$

Reynolds

$$\frac{\partial \bar{\rho}}{\partial t} + \nabla \cdot \left(\bar{\rho}\, \bar{\mathbf{v}} + \cancel{\overline{\rho' \mathbf{v}'}} \right) = 0$$

$$\bar{p} \cong R_0 \left(\bar{\rho}\, \overline{T_v} + \cancel{\overline{\rho' T_v'}} \right)$$

$$\frac{1}{2} \overline{\rho\, \mathbf{v}^2} = \frac{1}{2} \bar{\rho}\, \bar{\mathbf{v}}^2 + \frac{1}{2} \bar{\rho}\, \overline{\mathbf{v}'^2} + \bar{\mathbf{v}}\, \cancel{\overline{\rho' \mathbf{v}'}} + \frac{1}{2} \cancel{\overline{\rho' \mathbf{v}'^2}}$$

$$\frac{d\bar{\varphi}}{dt} = \frac{\partial \bar{\varphi}}{\partial t} + \bar{\mathbf{v}} \cdot \nabla \bar{\varphi} + \cancel{\overline{\mathbf{v}' \cdot \nabla \varphi'}}$$

Local similarity functions for the ASL

Monin – Obukhov (complete) similarity hypotheses

$$\frac{\kappa\,(z-d)}{u_*}\frac{\partial\hat{u}}{\partial z}=\Phi_m(\zeta)$$ **momentum**

$$\frac{\kappa\,(z-d)}{\Theta_*}\frac{\partial\hat{\Theta}}{\partial z}=\Phi_h(\zeta)$$ **sensible heat**

$$\frac{\kappa\,(z-d)}{q_*}\frac{\partial\hat{q}}{\partial z}=\Phi_q(\zeta)$$ **water vapor**

Obukhov number:

$$\zeta=\frac{z}{L}\begin{Bmatrix}<\\=\\>\end{Bmatrix}0$$

Obukhov stability length:

$$L=\frac{u_*^{\,2}}{\kappa\,\frac{g}{\Theta_m}\left(\Theta_*+0.608\,\Theta_m\,q_*\right)}$$

Vertical profile functions

(Panofsky, 1963)

$$\hat{q}(z_2) - \hat{q}(z_1) = \frac{q_*}{\kappa}\left(\ln\frac{z_2 - d}{z_1 - d} - \Psi_q(\zeta_2, \zeta_1)\right)$$

$$\hat{u}(z_2) - \hat{u}(z_1) = \frac{u_*}{\kappa}\left(\ln\frac{z_2 - d}{z_1 - d} - \Psi_m(\zeta_2, \zeta_1)\right)$$

$$\hat{\Theta}(z_2) - \hat{\Theta}(z_1) = \frac{\Theta_*}{\kappa}\left(\ln\frac{z_2 - d}{z_1 - d} - \Psi_h(\zeta_2, \zeta_1)\right)$$

Integral similarity functions for the ASL:

$$\Psi_{q,m,h}(\zeta_2, \zeta_1) = \int_{\zeta_1}^{\zeta_2} \frac{1 - \Phi_{q,m,h}(\zeta)}{\zeta}\, d\zeta$$

Customary assumptions for stable stratification

$$\zeta \geq 0$$

$$\Phi_m(\zeta) = 1 + \gamma_1 \zeta$$

Čalikov (1968), Webb (1970), Businger et al. (1971)

$$\Phi_q(\zeta) = \Phi_h(\zeta) = \Phi_m(\zeta)$$

Webb (1970)

$$\Psi_{q,m,h}(\zeta_2, \zeta_1) = -\gamma_1 \int_{\zeta_1}^{\zeta_2} d\zeta = -\gamma_1 (\zeta_2 - \zeta_1)$$

Logarithmic-linear profiles

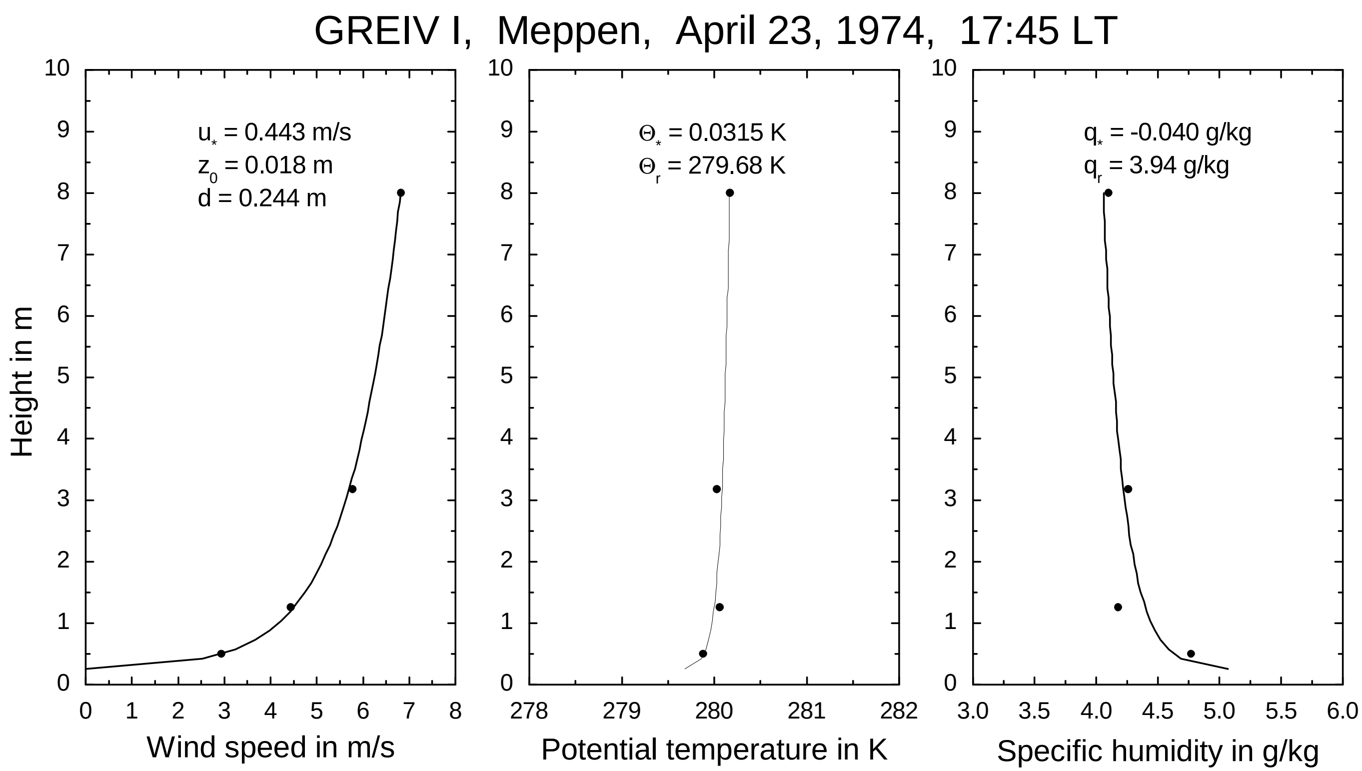
$$\hat{q}(z_2) - \hat{q}(z_1) = \frac{q_*}{\kappa}\left(\ln \frac{z_2 - d}{z_1 - d} + \gamma_1(\zeta_2 - \zeta_1) \right)$$

$$\hat{u}(z_2) - \hat{u}(z_1) = \frac{u_*}{\kappa}\left(\ln \frac{z_2 - d}{z_1 - d} + \gamma_1(\zeta_2 - \zeta_1) \right)$$

$$\hat{\Theta}(z_2) - \hat{\Theta}(z_1) = \frac{\Theta_*}{\kappa}\left(\ln \frac{z_2 - d}{z_1 - d} + \gamma_1(\zeta_2 - \zeta_1) \right)$$

Conventional Parameterization

“stable stratification”



Kramm & Herbert (2009)

Assuming d = 0 and $z_1 = z_0$ yields:

$$\hat{q}(z_2) = q(z_s) + \frac{q_*}{\kappa}\left(\kappa B_q^{-1} + \ln\frac{z_2}{z_0} + \gamma_1(\zeta_2 - \zeta_0)\right)$$

$$\hat{u}(z_2) = \frac{u_*}{\kappa}\left(\ln\frac{z_2}{z_0} + \gamma_1(\zeta_2 - \zeta_0)\right)$$

$$\hat{\Theta}(z_2) = T(z_s) + \frac{\Theta_*}{\kappa}\left(\kappa B_h^{-1} + \ln\frac{z_2}{z_0} + \gamma_1(\zeta_2 - \zeta_0)\right)$$

z_0 = roughness length for momentum

B_q, B_h = sublayer Stanton (Dalton) number

Sublayer Stanton (Dalton) number is positive-definite.

Parameterization for the sublayer Stanton (Dalton) numbers
(Garratt and Hicks, 1973)

$$\kappa B_h^{-1} = \frac{\hat{\Theta}_0 - T_s}{\Theta_*} = \ln \frac{z_0}{z_h}$$

$$\kappa B_q^{-1} = \frac{\hat{q}_0 - q_s}{q_*} = \ln \frac{z_0}{z_q}$$

$$z_h = \frac{\alpha}{\kappa u_*}$$ **roughness length for heat**

$$z_q = \frac{D_q}{\kappa u_*}$$ **roughness length for water vapor**

$$\hat{q}(z_2) = q(z_s) + \frac{q_*}{\kappa}\left(\ln\frac{z_2}{z_q} + \gamma_1(\zeta_2 - \zeta_0)\right)$$

$$\hat{u}(z_2) = \frac{u_*}{\kappa}\left(\ln\frac{z_2}{z_0} + \gamma_1(\zeta_2 - \zeta_0)\right)$$

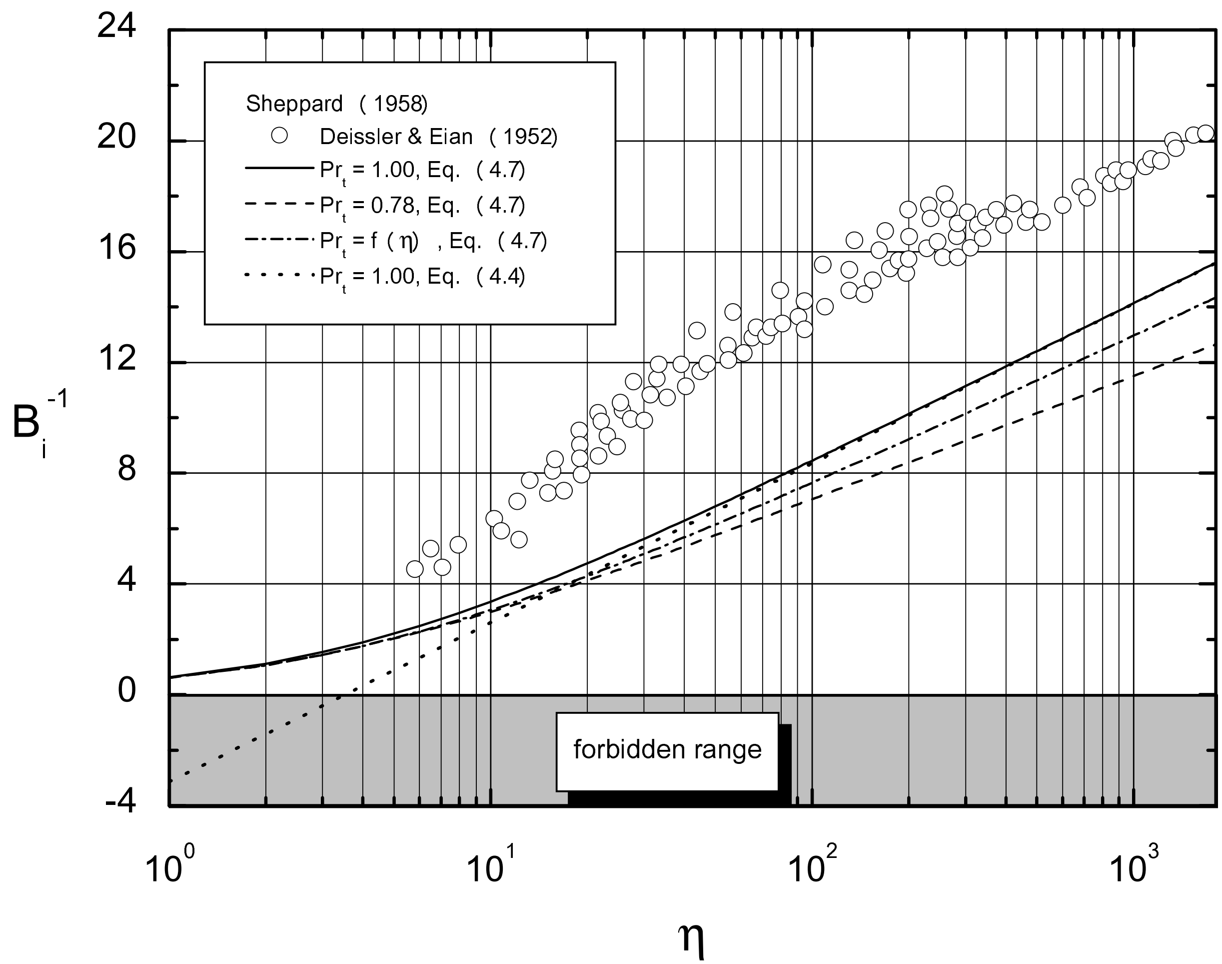
$$\hat{\Theta}(z_2) = T(z_s) + \frac{\Theta_*}{\kappa}\left(\ln\frac{z_2}{z_h} + \gamma_1(\zeta_2 - \zeta_0)\right)$$

$$z_0 \le z_{h,q} \quad \Rightarrow \quad B_{h,q} \le 0$$

$$\kappa B_h^{-1} = \ln\frac{z_0}{z_h} \qquad\qquad \kappa B_q^{-1} = \ln\frac{z_0}{z_q}$$

$$\kappa B_h^{-1} = \frac{\hat{\Theta}_0 - T_s}{\Theta_*} = \ln\left(1 + \frac{z_0}{z_h}\right)$$

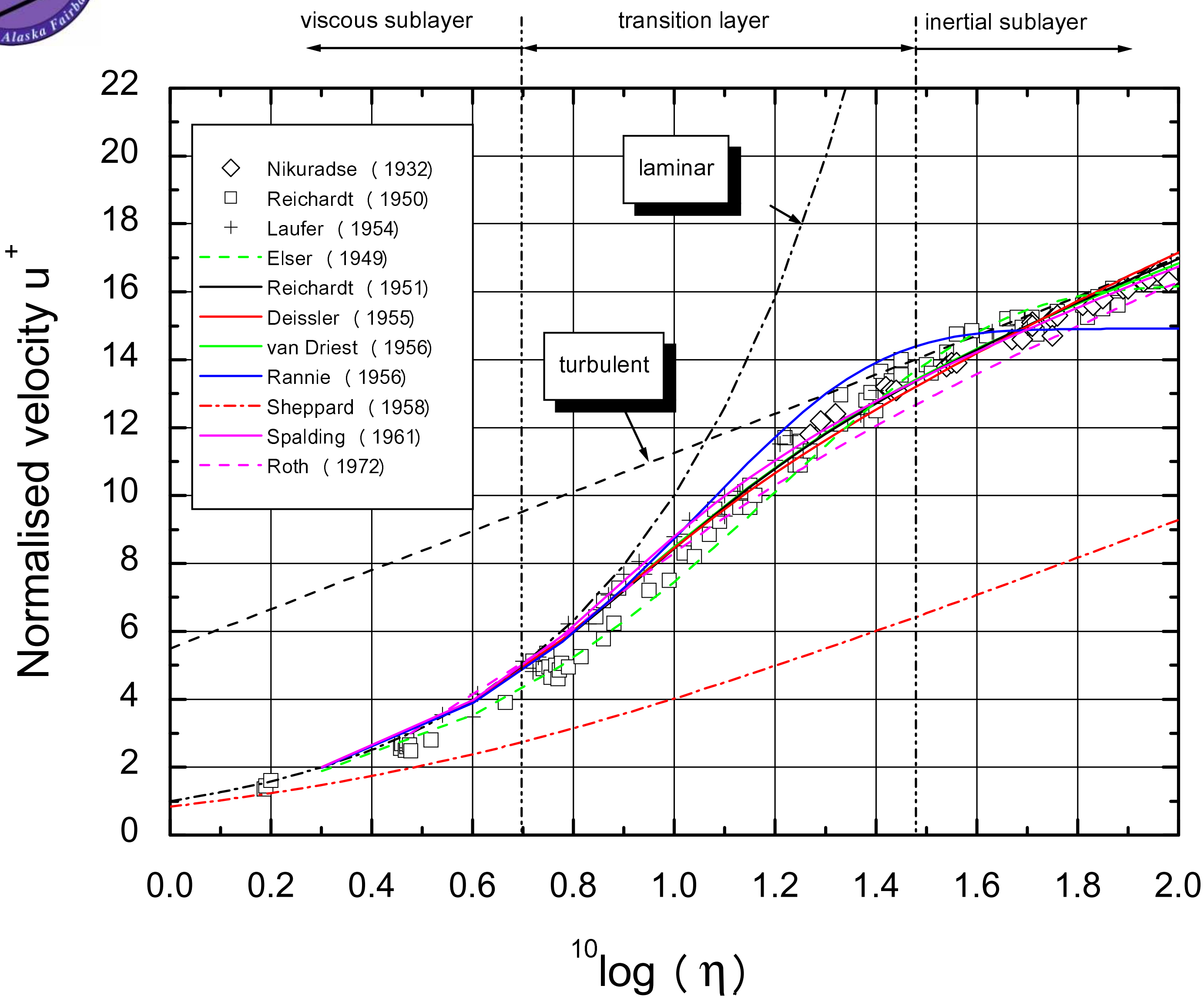
Sheppard (1958)



$$\eta = \frac{u_* z}{\nu}$$

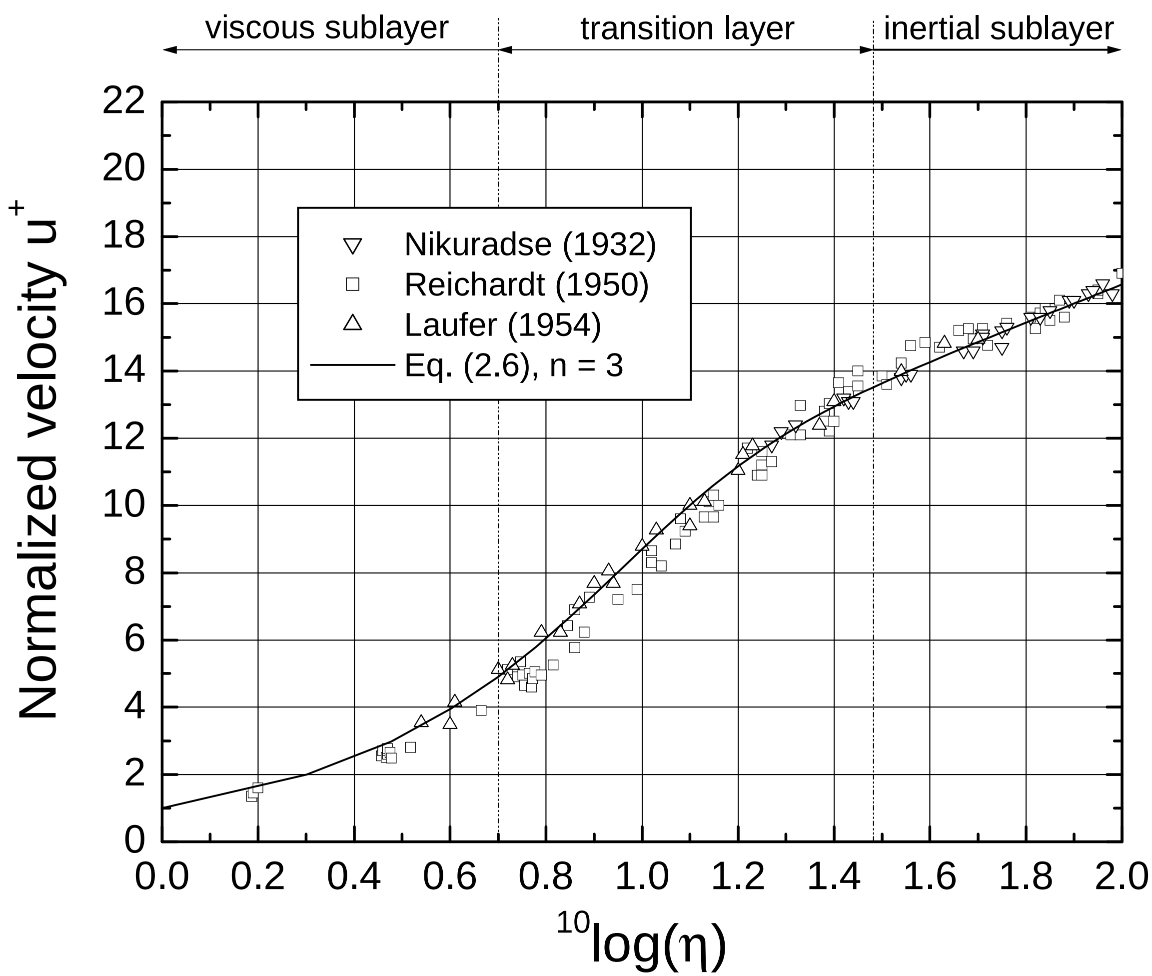
Kramm, Dlugi, and Mölders (2002)

$$u^+ = \frac{\hat{u}}{u_*}$$

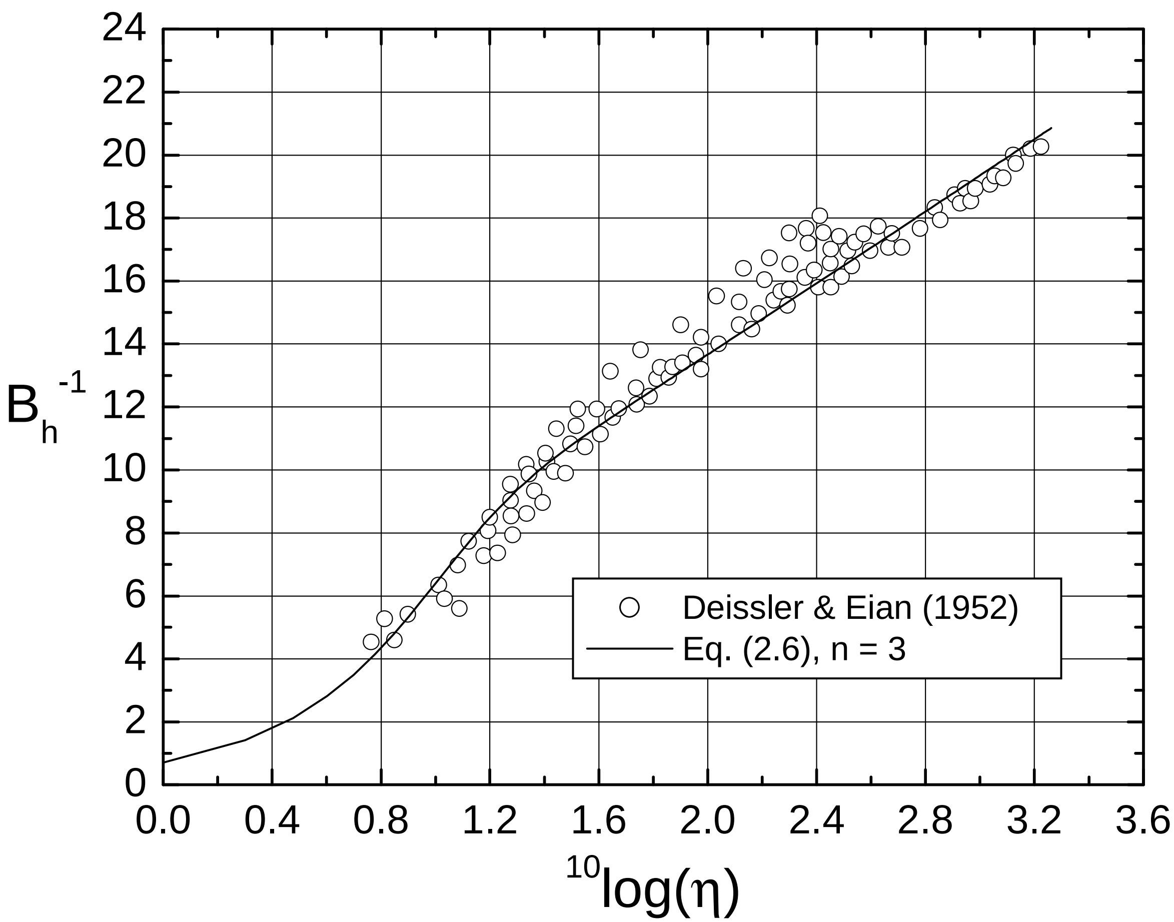


Kramm, Dlugi, and Mölders (2002)

$$\frac{K_m}{\nu} = \frac{\eta_D \, \eta^{\,n}}{\left(1 + \left(\frac{\eta_D}{\kappa}\right)^{\frac{n}{n-1}} \eta^{\,n}\right)^{\frac{n-1}{n}}}$$



Kramm and Mölders (2005)



Kramm and Mölders (2005)

Richardson Number vs. Obukhov Number

$$\mathrm{Ri} = \frac{\Phi_h(\zeta)}{(\Phi_m(\zeta))^2}\,\zeta$$

gradient-Richardson number

$$\mathrm{Ri} = \frac{\zeta}{1+\gamma_1\,\zeta}$$

or

$$\zeta = \frac{\mathrm{Ri}}{1-\gamma_1\,\mathrm{Ri}}$$

For $\gamma_1 = 5$, Ri has to satisfy the requirement Ri < 0.2

$$\mathrm{Ri} = \frac{\zeta}{1+\gamma_3\,\zeta}\left(1-\frac{0.26}{1+\gamma_3\,\zeta}\right)$$

Businger et al. (1971)

$$\mathrm{Ri} = \mathrm{Pr}_t \ \mathrm{Ri}_f$$

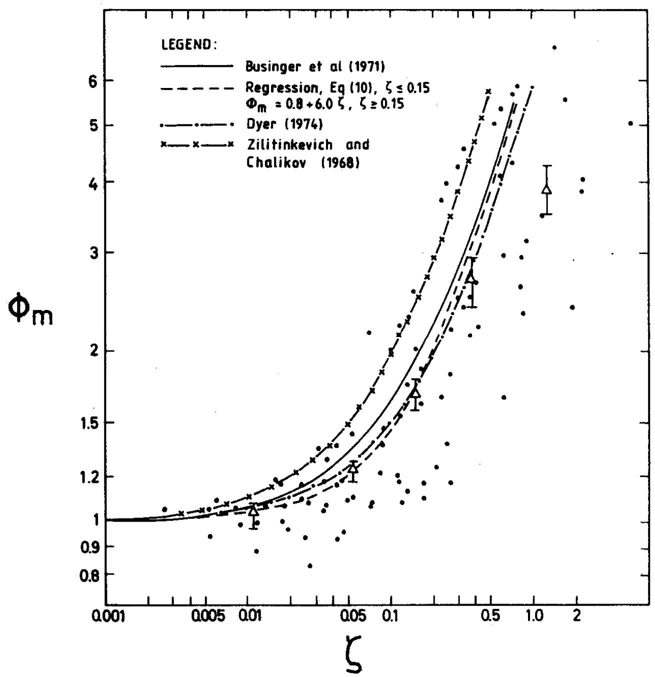
$$\mathrm{Pr}_t = \frac{\Phi_h(\zeta)}{\Phi_m(\zeta)} \cong 1$$

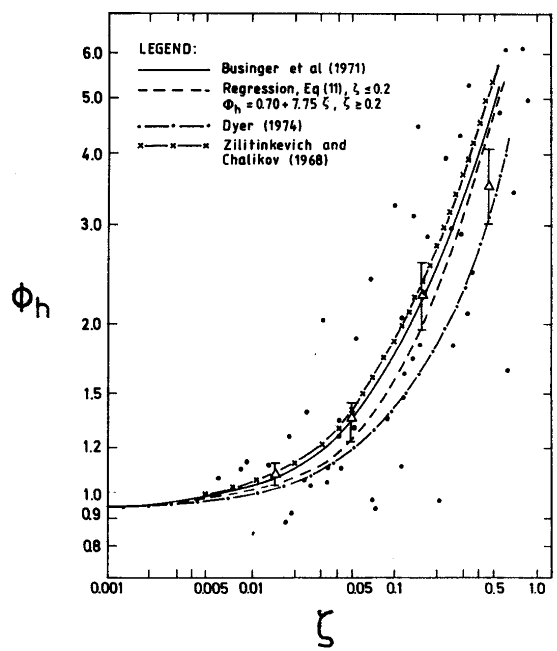
Pr_t = "turbulent" Prandtl number

Ri_f = flux-Richardson number

$$\gamma_1 = \frac{1}{\zeta\left(\frac{1}{\mathrm{Ri}} - 1\right)} \geq 0$$

not a constant





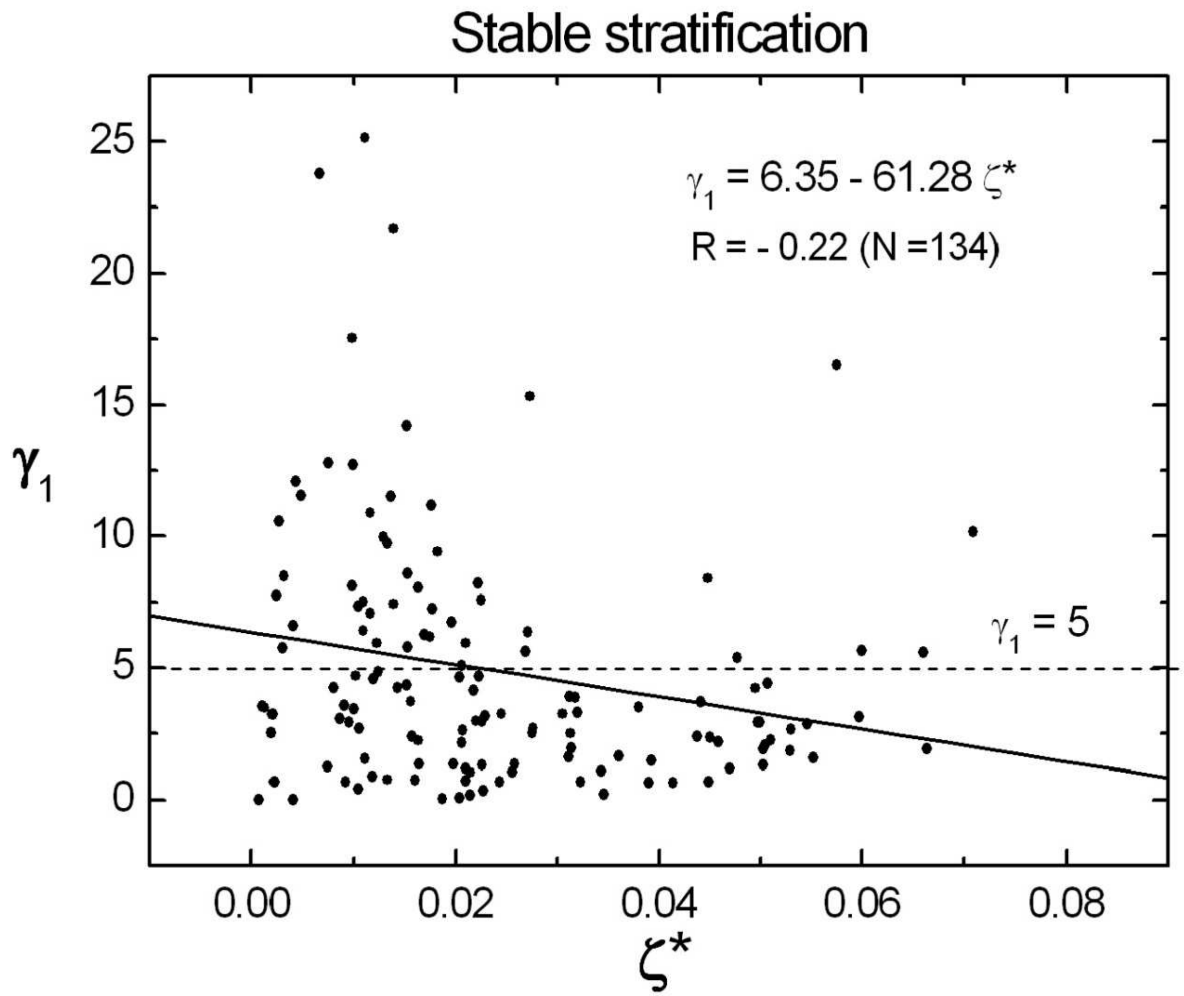
Source: Högström (1988)

$$\gamma_1 = \frac{1}{\zeta\left(\frac{1}{\mathrm{Ri}} - 1\right)} \geq 0$$

$$\mathrm{Ri}_{cr} \cong \mathrm{Ri}_{f,cr} = 0.25$$

$$\gamma_1 = 1/(3\,\zeta)$$

$$\gamma_1 \leq 1/6$$



$$\zeta^* = z^*/L \qquad z^* = \sqrt{z_N \; z_{N+1}}$$

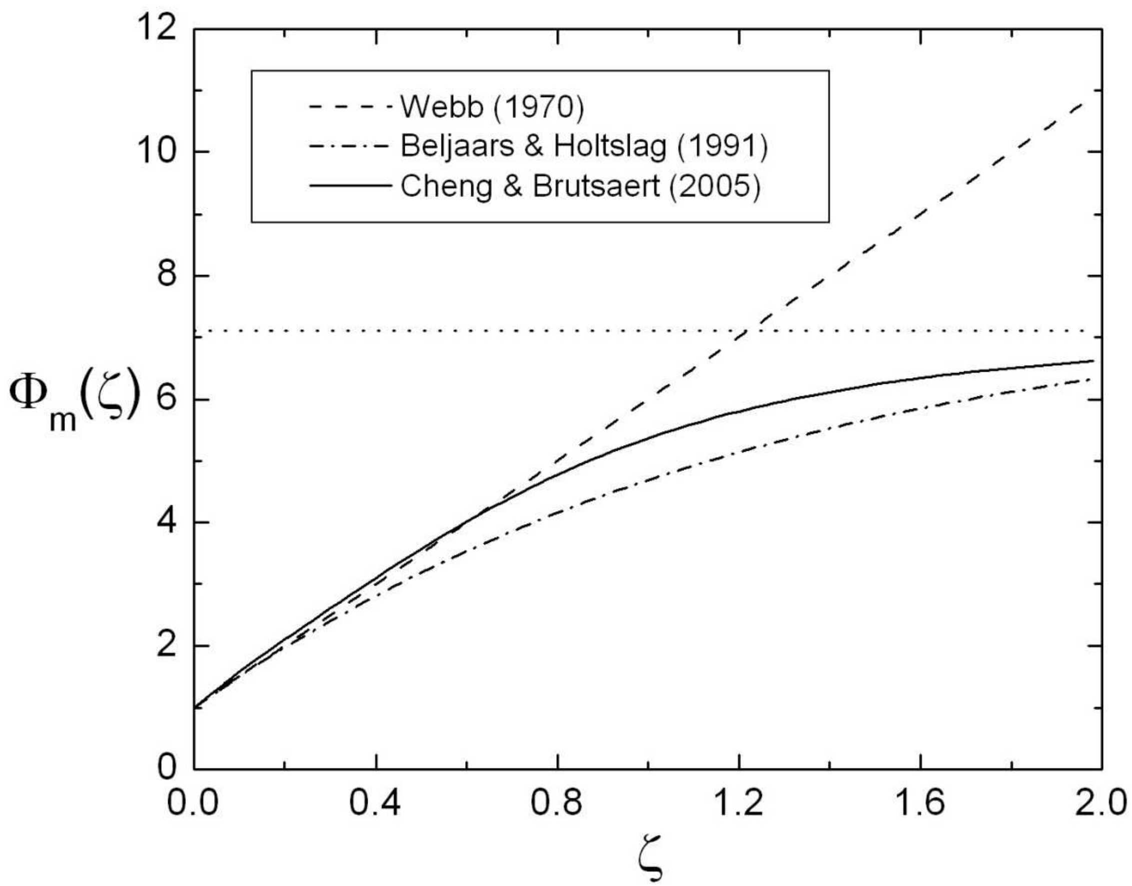
Kramm et al. (1996)

Cheng and Brutsaert (2005)

$$\Phi_m(\zeta) = 1 + \gamma_6 \left(\frac{\zeta + \zeta^{\gamma_7} \left(1 + \zeta^{\gamma_7}\right)^{\frac{1-\gamma_7}{\gamma_7}}}{\zeta + \left(1 + \zeta^{\gamma_7}\right)^{\frac{1}{\gamma_7}}} \right)$$

$$\Phi_h(\zeta) = 1 + \gamma_8 \left(\frac{\zeta + \zeta^{\gamma_9} \left(1 + \zeta^{\gamma_9}\right)^{\frac{1-\gamma_9}{\gamma_9}}}{\zeta + \left(1 + \zeta^{\gamma_9}\right)^{\frac{1}{\gamma_9}}} \right)$$

$$\gamma_6 = 6.1 \qquad \gamma_7 = 2.5 \qquad \gamma_8 = 5.3 \qquad \gamma_9 = 1.1$$



$$\frac{\kappa\left(z-d\right)}{u_*}\frac{\partial \hat{u}}{\partial z}=\text{const.}$$

leads to a logarithmic wind profile even for strongly stable stratification

Laminar (viscous) flow

$$\frac{\partial \hat{u}}{\partial \eta} = u_*$$

expected are linear profiles

$$\frac{\partial \hat{\Theta}}{\partial \eta} = \Pr \Theta_*$$

$$\frac{\partial \hat{q}}{\partial \eta} = Sc_q q_*$$

Complete similarity must not be expected in the case of strongly stable stratification.

Kramm and Herbert (2009)

Summary and Conclusions

- The parameterization schemes can reasonably be applied to measured vertical profile data.

- The concept of the roughness length for scalar quantities is physically inadequate.

- The conventional Monin-Obukhov similarity hypotheses are based on complete similarity, but in the case of strongly stable stratification complete similarity must not be expected.